AD-A038 937 IOWA STATE UNIV AMES DEPT OF MATHEMATICS F/6 12/2
ALGEBRAIC TESTS FOR FUNCTIONAL CONTROLLABILITY OF FUNCTIONAL DI--ETC(U)
APR 77 R TRIGGIANI AF-AFOSR-3038-76
UNCLASSIFIED AFOSR-TR-77-0545 NL

| OF |

AD A038937

END

DATE FILMED 5-77

AFOSR - TR - 77 - 0545

ADA038937

See 1473

AIR FORCE OF SCIENTIFIC RESEARCH

Directorate of Mathematical and Information Service

Program	Applied Mathematics (control theory)
Attention:	Capt. Charles L. Nefzger
From:	R. Triggiani, Mathematics Department Iowa State University, Ames, Iowa 50011 Telephone (515)294-8168
Grant No.:	AFOSR-76-3038
Research Title:	Algebraic Tests for Functional Controllability of Functional Differential Equation Systems.
Principal Investigator:	R. Triggiani

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DDC
This technical report has been reviewed and is
approved for public release IAW AFR 190-12 (7b).
Distribution is unlimited.
A. D. BLOSE
Technical Information Officer

Summary of Main Results on Function Space Controllability and Stabilizability Obtained in Original Proposal

Controllability properties of linear retarded control systems of the type

$$\dot{y}(t) = A_0y(t) + A_1y(t-h) + Bu(t) \tag{A.1}$$

where h is a positive constant, $y \in R^n$, $u \in R^m$, and A_0, A_1, and B are matrices of appropriate dimensions, were investigated. The approach followed consists in replacing (A.1) by its abstract representation given by

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) \tag{A.2}$$

Here x belongs to the Hilbert space $R^n \times L_2[[-h,0],R^n]$, denoted shortly by M_2, $\tilde{A}$ is a certain first order differential operator generating a C_0-semigroup, and $\tilde{B}$ is a certain operator with finite dimensional range.

More specifically, the following concepts were investigated:

1) <u>M_2-approximate controllability of (A.1)</u> (Loosely: steer the solution of (A.1) arbitrarily close to a preassigned pair consisting of an $L_2[[-h,0],R^n]$-target function and an R^n-target point.)

2) <u>L_2-approximate controllability of (A.1)</u> (Loosely: steer the solution of (A.1) arbitrarily close to a preassigned $L_2[[-h,0],R^n]$-target function.)

3) R^n(or Euclidean) controllability of (A.1) (Loosely: steer the solution of (A.1) to hit exactly a preassigned R^n-target point.)
(For precise definitions, cf. Original Proposal)

4) Spectral controllability of (A.1) described below
Consider the equation (A.2) - the abstract version of (A.1) - and

Consider the equation (A.2) - the abstract version of (A.1) - and

project it onto each of the countably many generalized eigenspaces associated to the operator $\tilde{A}$. Since each such eigenspace is finite dimensional, each projection is a linear ordinary (finite dimensional) differential equation. Then (A.1) is called 'spectrally controllable' in case all its associated projections onto the generalized eigenspaces are controllable (in the usual, finite dimensional sense).

One importance of the concept of spectral controllability is that it provides a sufficient condition (weaker than M_2-approximate controllability in fact!) for feedback stabilizability of (A.1) with arbitrarily prefixed exponential decay; this means that there exists a linear operator $F: M_2 \to R^m$ such that the feedback input

$$u(t) = F(y(t), y(+\phi)), \quad -h \leq \phi \leq 0$$

once substituted in (A.1), makes (A.1) a globally asymptotically stable linear retarded delay equation, bounded above by $M_\delta e^{-\delta t}$, $t \geq 0$, for some $M_\delta > 0$ and with δ arbitrarily prescribed positive constant.

Our declared goal in the Original Proposal was to characterize the above concepts of controllability in terms of easy-to-check tests involving only the data defining (A.1), i.e., the matrices A_0, A_1, B and the constant h. Our declared approach was to use, as a starting point, an abstract characterization for controllability of general control systems in Banach spaces, as it applies to the infinite dimensional version (A.2) of the system (A.1). (See Original Proposal). (Such abstract characterization is a combination of results due to Fattorini and myself.) In previous papers, I had successfully applied such abstract characterization to derive easy to check tests for controllability of both parabolic and hyperbolic partial differential equations (P.D.E.) and other

infinite dimensional systems. Manitius and I have finally succeeded in applying such abstract characterizations to derive simple tests also for the above controllability concepts of the delay-equations (A.1). As a consequence, it is possible now to treat the controllability properties of various dynamical systems as parabolic, hyperbolic P.D.E., and delay-equations (plus other distributed parameter systems) in a mathematically unified way within the same framework of control systems defined on Banach spaces (semigroup theory of operators). Following the indicated unified approach, Manitius and I have, in one stroke, solved the L_2- and M_2- approximate controllability problems, which were completely open; provided a new treatment for rederiving already known tests for Euclidean controllability; derived new conditions for spectral controllability (and hence stabilizability) much easier to handle than those previously known; brought to light an interesting link between pointwise degeneracy and lack of L_2-approximate controllability; and clarified the relationship between L_2-approximate controllability and Popov reachability (see Original Proposal).

While detailed results are to be found in our paper, which are listed at the end, I briefly list some of our main findings:

1. Necessary conditions for L_2- (and M_2-) approximate controllability of (A.1) stated in terms of the rank of a certain polynomial $n \times nm$ matrix $P(\lambda)$, *which is easily computable from the original system matrices* A_0, A_1, and B. ($P(\lambda)$ does not depend on the delay h).

2. Necessary and sufficient conditions for L_2- and M_2-approximate controllability of (A.1) that reduce to the algebraic question on whether a system of linear homogeneous equations has a non-zero solution.

3. Sufficient conditions for M_2-approximate controllability of (A.1) for all values of $h>0$ stated directly in terms of the original matrices

A_0, A_1, and B. (For $n \geq 3$, the particular value of the delay h is shown through examples to be crucial even for L_2-approximate controllability.)

4. A new treatment of Euclidean controllability written the same general unified approach described above.

5. Simple criteria for spectral controllability, hence for feedback stabilizability with arbitrarily prefixed exponential decay rate. These improve on previously known results in that they do not require knowledge of the eigenvalues of $\tilde{A}$ (countably many, in general) but only (when $m = 1$) the roots of the polynomial det $P(\lambda)$.

6. A link between pointwise degeneracy of (A.1) and lack for it of L_2-approximate controllability.

7. L_2-approximate controllability implies Popov reachability but not conversely.

Moreover, many examples complement the theoretical results and show the various links (or lack thereof) of the different concepts involved. Mathematically, our analysis is carried out through three distinct stages:

i. Functional analysis and semigroup of operator theory,

ii. Theory of entire functions, and

iii. Matrix theory and linear algebra.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(19) REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER (18) AFOSR - TR-77-0545

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle) (6) ALGEBRAIC TESTS FOR FUNCTIONAL CONTROLLABILITY OF FUNCTIONAL DIFFERENTIAL EQUATION SYSTEMS.

5. TYPE OF REPORT & PERIOD COVERED (9) Interim rept.,

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s) (10) R. Triggiani

8. CONTRACT OR GRANT NUMBER(s) (15) ✓ AF- AFOSR 76-3038-76 new

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Iowa State Univ~~ersity~~
Mathematics Department
Ames, Iowa 50011

410171 New

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
61102F
(16) 2304/A1 (17) A1

11. CONTROLLING OFFICE NAME AND ADDRESS
Air Force Office of Scientific Research/NM
Bolling AFB DC 20332

12. REPORT DATE (11) Apr 1977

13. NUMBER OF PAGES 5

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 7p.

15. SECURITY CLASS. (of this report) UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Results obtained during this period of research includ solving two approximate controllability problems, which were previously open. New treatments were provided for rederiving already known tests for Euclidean controllability. New conditions were derived for spectral controllability (and hence stabilizability) much easier to handle than those previously known. An interesting link was brought to light between point wise degeneracy and the lack of approximate controllability in a Hilbert defined approximate controllability and Popov reachability was clarified.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

6B

Publications originated from research activity under the grant

1) New Results on functional controllability of time-delay systems* (with A. Manitius) — Proceedings of the Conference on System Theory, Johns Hopkins University, March 31, April 1-2, 1976.

2) Sufficient conditions for function space controllability and feedback stabilizability of linear retarded systems (with A. Manitius) — Proceedings of the 1976 Decision and Control Conference, Florida, Dec. 1976. Also to appear, in a shorter version, in the Proceedings of the International Conference on Dynamical Systems, Institute of Applied Mathematics, University of Florida, Gainesville, March 1976, published by Academic Press.

3) Function space controllability of linear retarded systems: A derivation from abstract operator conditions (with A. Manitius) — Report of 114 pages, submitted.

Moreover, I have completed and revised the following paper:

4) A note on the lack of exact controllability for mild solutions in Banach space — To appear in SIAM J. Control and Optimization, May 1977.